Department of Mathematics and Computer Science
College of Science
University of the Philippines Baguio

Math 133 Syllabus

A. COURSE DETAILS

Course Name	**Introduction to Functional Analysis**	
Course Description	This is an introductory course designed to study vector spaces equipped with topologies induced by norms and inner products. It will cover topics such as normed, inner-product, Banach and Hilbert spaces, duality, orthogonal decompositions, orthonormal bases, basic Fourier series and Banach algebras.	
Credit Units	3 units (Lecture)	
Prerequisite	Math 130	
Requirements	Two Long Examinations	50%
	Problem Sets	30%
	Final Examination	20%
Passing Grade	60%	

B. COURSE OUTCOMES

At the end of the course, the students must be able to:

CO1 Demonstrate understanding of metric spaces through examples.

CO2 Discuss the process of completing metric spaces.

CO3 State and prove the contraction principle and demonstrate its application to the local-in-time existence and uniqueness of solutions to ordinary differential equations.

CO4 Give fundamental examples of normed spaces and Banach spaces.

CO5 Demonstrate the completeness or incompleteness of normed vector spaces.

CO6 Identify the dual of fundamental sequence spaces and Lebesgue spaces.

CO7 Prove the completeness of the space of bounded linear operators from a normed space to a Banach space.

CO8 State the Hahn-Banach Theorem and cite some consequences.

CO9 Give fundamental examples of Hilbert spaces.

CO10 Illustrate the importance of orthogonality in approximation and optimization theory.

CO11 State the Lax-Milgram Lemma and compare the proofs between a fixed-point-based argument and the Riesz Representation Theorem.

CO12 Calculate the Fourier series expansion of a periodic piecewise continuous function.

CO13 Define a Banach algebra and discuss few examples.

C. COURSE OUTLINE

Timeline	Course Outcome	Topics	Assessment Tools
Week 1-2	CO1 CO2 CO3	**I. Preliminaries** - Metric spaces, convergence, Cauchy sequence, completeness, completion of metric spaces - Banach fixed point theorem, application to existence theory for ordinary differential equations	Problem Set Written Exam
Week 3-5	CO4 CO5	**II. Normed and Banach spaces** - Vector spaces, semi-normed spaces, normed spaces, Banach spaces, equivalence of norms - sequence spaces, space of bounded functions, space of continuously differential functions, space of Hoelder continuous functions - Introduction to Lebesgue spaces	Problem Set Written Exam
Week 6-8	CO6 CO7 CO8	**III. Bounded linear operators** - Bounded linear operators, bounded linear functionals - Dual spaces, Riesz-Fischer Theorem, reflexive spaces - Hahn-Banach Theorem **FIRST LONG EXAMINATION**	Problem Set Written Exam
Week 9-13	CO9 CO10 CO11	**IV. Inner Product and Hilbert Spaces** - Inner product spaces, Hilbert spaces - Orthogonality - Riesz Representation Theorem - Lax-Milgram Lemma and its extensions - Variational inequalities and proximity problems	Problem Set Written Exam
Week 14-16	CO12 CO13	**V. Basic Fourier Analysis and Banach Algebras** - Orthonormal bases, Bessel's inequality, Parseval's identity - Fourier series, Riesz bases, nonharmonic Fourier analysis - Introduction to Banach algebras **SECOND LONG EXAMINATION**	Problem Set Written Exam
		FINAL EXAMINATION	

D. REFERENCES

1. J.B. Conway, *A Course in Functional Analysis*, *4th ed.,* Springer, 1987.

2. N. Dunford and J.T. Schwartz, *Linear Operators I and II*, *2nd ed.,* Interscience Publishers, 1988.

3. E. Hille and R.S. Phillips, *Functional Analysis and Semi-Groups*, American Mathematical Society, 1957.

4. E. Kreyszig, *Introductory Functional Analysis with Applications*, John Wiley & Sons, 1978.

5. P.D. Lax, *Functional Analysis*, Wiley, 2002.

6. M. Miklavcic, *Applied Functional Analysis and Partial Differential Equations*, World Scientific, 1998.

7. W. Rudin, *Functional Analysis*, *2nd ed.*, McGraw-Hill, 2006.

8. B.P. Rynne and M.A. Youngson, *Linear Functional Analysis*, *2nd ed.,* Springer-Verlag, 2008.

9. A.H. Siddiqi, *Functional Analysis and Applications*, Springer, 2018.

10. E.M. Stein and R. Shakarchi, *Functional Analysis: Introduction to Further Topics in Analysis*, Princeton University Press, 2011.

E. CLASS RULES

1. The University rule on class attendance (Article 346 of the University Code) shall be strictly enforced.

2. If a student misses a short quiz, his/her grade in that quiz is zero. If a student misses a long examination for a valid reason (this requires documentation), his/her final grade in the final exam will also account as his/her grade for the missed exam. This applies to no more than one long exam missed. A student who fails to take any examination for invalid reasons will get a grade of 0% for that exam.

3. Cheating, in any form, will not be tolerated.