



**Department of Mathematics and Computer Science**  
 College of Science  
 University of the Philippines Baguio

## SYLLABUS

### A. COURSE DETAILS

Course Number:	Math 123
Course Name:	Elementary Number Theory
Course Description:	This course covers the algebra of complex numbers, the topology of the complex plane, differentiability of complex functions, harmonic functions, the Cauchy integral theorem and formula, Taylor and Laurent series, the residue theorem and applications.
Credit Units:	3 units (Lecture)
Prerequisite:	Math 29 or CMSC 55
Requirements:	Three Long Examinations 54%
	Quizzes, Homework, Problem Sets, Reporting 26%
	Final Examination 20%
Passing Grade:	60%

### B. COURSE OUTCOMES

At the end of this course, the students must be able to

- CO1: Describe how the method of mathematical induction works;
- CO2: Recognize the binomial coefficients;
- CO3: List some divisibility rules;
- CO4: Recognize the Euclidean algorithm and its role in the study of greatest common divisors and linear Diophantine equations;
- CO5: Define what a prime number is;
- CO6: Recall the Fundamental Theorem of Arithmetic;
- CO7: Compare various methods of factorization;
- CO8: State the Chinese remainder theorem;
- CO9: Discuss the concept of congruence;
- C10: Identify linear congruence and systems of congruences;
- C11: Discuss applications of congruences;
- C12: Explain well known theorems, such as Wilson's Theorem, Fermat's theorems and

Euler theorem;

C13: Classify pseudoprimes;

C14: Demonstrate how the Euler-phi function works;

C15: Use number-theoretic functions in determining the number and sum of divisors of a given number;

C16: Analyze problems that involve perfect numbers and Mersenne primes;

C17: Differentiate various methods that are used in encrypting and deciphering messages;

C18. Apply quadratic reciprocity law in solving equations;

C19. Check whether the numbers are quadratic residues or quadratic non-residues modulo  $n$ ;

C20. Solve problems on primitive roots; and

C21. Research on topics like Pythagorean triples, Fermat's last theorem, sums of squares and Pell's equation.

### C. COURSE OUTLINE

Timeline	Course Outcome	Topics	Assessment Tools
Week 1-2	CO1 CO2 CO3	<b>The Integers</b> <ul style="list-style-type: none"> <li>• Mathematical induction</li> <li>• Binomial coefficients</li> <li>• Divisibility</li> </ul>	<b>Boardwork Assignment Quiz</b>
	CO4 CO5 CO6 CO7	<b>Greatest Common Divisor and Prime Factorization</b> <ul style="list-style-type: none"> <li>• Greatest common divisor</li> <li>• The Euclidean algorithm</li> <li>• The fundamental theorem of arithmetic</li> <li>• Fermat numbers and factorization methods</li> <li>• Linear Diophantine equations</li> </ul> <b>First Long Examination</b>	<b>Boardwork Assignment Quiz Exam</b>

	<b>CO8</b> <b>CO9</b> <b>C10</b> <b>C11</b>	<p style="text-align: center;"><b>Congruences</b></p> <ul style="list-style-type: none"> <li>• Introduction to congruences</li> <li>• Linear congruences</li> <li>• The Chinese remainder theorem</li> <li>• Systems of linear congruences</li> </ul>	<b>Boardwork</b> <b>Assignment</b> <b>Quiz</b>
	<b>CO3</b>	<p style="text-align: center;"><b>Applications of Congruences</b></p> <ul style="list-style-type: none"> <li>• Divisibility tests</li> <li>• The perpetual calendar</li> </ul>	<b>Boardwork</b>
	<b>C12</b> <b>C13</b>	<p style="text-align: center;"><b>Some Special Congruences</b></p> <ul style="list-style-type: none"> <li>• Wilson's theorem and Fermat's little Theorem</li> <li>• Pseudoprimes</li> <li>• Euler's theorem</li> </ul>	<b>Quiz</b>
	<b>C14</b> <b>C15</b> <b>C16</b>	<p style="text-align: center;"><b>Multiplicative Functions</b></p> <ul style="list-style-type: none"> <li>• Euler's phi function</li> <li>• The sum and number of divisors</li> <li>• Perfect numbers and Mersenne primes</li> </ul> <p style="text-align: center;"><b>Second Long Examination</b></p>	<b>Boardwork</b> <b>Assignment</b> <b>Quiz</b>
	<b>C17</b>	<p style="text-align: center;"><b>Cryptology</b></p> <ul style="list-style-type: none"> <li>• Character ciphers</li> <li>• Block ciphers</li> <li>• Exponential ciphers</li> <li>• Public key cryptography</li> <li>• Knapsack ciphers</li> </ul>	<b>Assignment</b> <b>Reporting</b>
	<b>C19</b> <b>C20</b>	<p style="text-align: center;"><b>Primitive Roots</b></p> <ul style="list-style-type: none"> <li>• The order of an integer and primitive roots</li> <li>• Primitive roots for primes</li> <li>• Existence of primitive roots</li> </ul>	<b>Boardwork</b> <b>Assignment</b> <b>Quiz</b>
	<b>C18</b>	<p style="text-align: center;"><b>Quadratic Residues and Reciprocity</b></p> <ul style="list-style-type: none"> <li>• Quadratic residues and nonresidues</li> <li>• Quadratic reciprocity</li> </ul>	<b>Boardwork</b>

	<b>C21</b>	<b>Some Nonlinear Diophantine Equations (Optional)</b>	<b>Paper Exam</b>
		<ul style="list-style-type: none"> <li>• Pythagorean triples</li> <li>• Fermat's last theorem</li> <li>• Sums of squares</li> <li>• Pell's equation</li> </ul>	
<b>Third Long Examination</b>			
<b>FINAL EXAMINATION</b> <i>(as scheduled by the Office of the University Registrar)</i>			

#### D. REFERENCES

- Rosen, K.H., **Elementary Number Theory and Its Applications**, 5th Ed., Addison Wesley, 2005.
- Burton, D.M., **Elementary Number Theory**, Revised Printing, Wm. C. Brown Publishers, 1988.
- Schroeder, M.R., **Number Theory in Science and Communication**, 5th Ed., Springer-Verlag, 2009.
- Tattersall, J.J. **Elementary Number Theory in Nine Chapters**, 2nd Ed., Cambridge University Press, 2005.
- Niven, I.M. et. al., **An Introduction to the Theory of Numbers**, 5th Ed., Wiley, 1991

#### E. CLASS RULES

1. The University rule on class attendance (Article 346 of the University Code) shall be strictly enforced in the class.
2. If a student misses a short quiz, his/her grade in that quiz is zero. If a student misses a long exam for a valid reason (this requires documentation), his/her grade in the final exam will also account as his/her grade for the missed exam. This applies to no more than one long exam missed. A student who fails to take any examination for invalid reasons will get a grade of 0% for that exam.
3. Cheating, in any form, will not be tolerated.

**F. RUBRIC FOR ASSESSMENT**

**A. Problem Set**

**CRITERIA**

<b>0</b>	<b>Unacceptable</b>
<b>1</b>	<b>Poor</b>
<b>2</b>	<b>Basic</b>
<b>3</b>	<b>Acceptable</b>
<b>4</b>	<b>Exemplary</b>

**Interpretation of the Problem**

**30%** Incorrect interpretation of problem. A major misinterpretation of what is given or what is to be shown. There is at least some sign of relevant ideas regarding the problem. Correct but incomplete interpretation of the problem. May overlook significant details in the statement of the problem. Might be stated for indirect proof but a direct proof is given or vice-versa. Correct but with minor incorrect or unnecessary concepts for its solutions. Correct statement with the hypothesis (given) and conclusion (to show) clearly stated.

**Correctness of Proof**

**70%** Mainly incorrect consequences Improperly deduced from the given. Little or no sense of how to prove the result. Unconnected, mostly true statements properly deduced from the given. Listing facts without a sense of how to link them to get a correct proof. May just jump to the conclusion without justification. Statements linked into a reasonable (though perhaps misguided) attempt to prove the theorem. The proof may be left incomplete or may depend upon a major unjustified leap. A correct approach to proving the theorem is attempted. Some statements may be unjustified or improperly justified, but errors are minor and could be fixed without substantially changing the proof. A correct and complete proof is given. Some irrelevant information may be included, particularly on timed work where the student is unable to polish up the presentation.

**B. Reporting**

**Criteria Needs Improvement**

- 1 Satisfactory**
- 2 Good**
- 3 Exemplary**
- 4**

**Organization**

**10%** Audience cannot understand presentation because there is no sequence of information. Audience has difficulty following presentation because student jumps around. Student presents information in logical sequence which audience can follow. Student presents information in logical, interesting sequence which audience can follow.

**Content Knowledge**

**50%** Students shows no understanding of mathematical concepts within the presentation Students are visibly uncomfortable with the mathematical concepts of the presentation Students are at ease with the mathematical concepts of the presentation but lack a deep conceptual understanding Students demonstrate a complete and comprehensive understanding of the mathematical concepts in the presentation

**Visuals**

**10%** Students use no visuals Students occasionally use visuals that rarely support the presentation and audience understanding Students use visuals that are related to the presentation but did not completely support audience understanding The visuals used supported audience understanding

**Mechanics**

**10%** Students presentation contained four or more spelling, grammatical or mathematical errors Presentation had three spelling, grammatical or mathematical errors Presentation had no more than two spelling, grammatical or mathematical errors Presentation had no spelling, grammatical or mathematical errors

**Delivery**

**20%** Student mumbles, incorrectly pronounces terms, and speaks too quietly for students in the back of class to hear. Student incorrectly pronounces terms. Audience members have difficulty hearing presentation. Student's voice is clear. Student pronounces most words correctly. Student used a clear voice and correct, precise pronunciation of terms.



